A generalized reference-plane-based calibration method in optical triangular profilometry

Suochao Cui, [1,2] and Xiao Zhu[1,2*]

[1]*Institute of Optoelectronics Science and Engineering, Huazhong University of Science and Technology, 1037 Luoyu Road, Wuhan, 430074, China*
[2]*Wuhan National Laboratory for Optoelectronics, 1037 Luoyu Road, Wuhan, 430074, China*
**zx@mail.hust.edu.cn*

Abstract: In this paper, a generalized reference-plane-based calibration method is proposed in optical triangular profilometry by exploring projection ray tracing method and image ray tracing method. The pin-hole camera model is used to model the camera and the projector, and parallel planes model is used to model the reference and test planes. The camera, projector, and planes can be in arbitrary positions and arbitrary directions. The reciprocal of the height and the reciprocal of the phase shift (or pixel position vertical distance) are in linear relationship. Experiments are conducted to verify the proposed method.

OCIS codes: (120.2830) Height measurements; (120.5050) Phase measurement; (120.6650) Surface measurements, figure.

References and links

1. W. S. Zhou, and X. Y. Su, "A direct mapping algorithm for phase-measuring profilometry," J. Mod. Opt. **41**(1), 89–94 (1994).
2. L. Chen, and C. Quan, "Fringe projection profilometry with nonparallel illumination: a least-squares approach," Opt. Lett. **30**(16), 2101–2103 (2005).
3. L. Chen, and C. J. Tay, "Carrier phase component removal: a generalized least-square approach," J. Opt. Soc. Am. A **23**(2), 435–443 (2006).
4. H. Guo, H. He, Y. Yu, and M. Chen, "Least-squares calibration method for fringe projection profilometry," Opt. Eng. **44**(3), 033603 (2005).
5. H. Guo, M. Chen, and P. Zheng, "Least-squares fitting of carrier phase distribution by using a rational function in fringe projection profilometry," Opt. Lett. **31**(24), 3588–3590 (2006).
6. B. A. Rajoub, M. J. Lalor, D. R. Burton, and S. A. Karout, "A new model for measuring object shape using non-collimated fringe-pattern projections," J. Opt. A, Pure Appl. Opt. **9**(6), S66–S75 (2007).
7. Z. Wang, H. Du, and H. Bi, "Out-of-plane shape determination in generalized fringe projection profilometry," Opt. Express **14**(25), 12122–12133 (2006).
8. H. Du, and Z. Wang, "Three-dimensional shape measurement with an arbitrarily arranged fringe projection profilometry system," Opt. Lett. **32**(16), 2438–2440 (2007).
9. Z. Wang, H. Du, S. Park, and H. Xie, "Three-dimensional shape measurement with a fast and accurate approach," Appl. Opt. **48**(6), 1052–1061 (2009).
10. A. Asundi, and Z. Wensen, "Unified calibration technique and its applications in optical triangular profilometry," Appl. Opt. **38**(16), 3556–3561 (1999).
11. J. Heikkila, and O. Silven, "Calibration Procedure for short focal length off-the-shelf CCD cameras," in *Proceedings of IEEE Conference on Computer Vision and Pattern Recognition* (Vienna, Austria, 1996), pp. 166–170.
12. Z. Zhang, "A flexible new technique for camera calibration," IEEE Trans. Pattern Anal. Mach. Intell. **22**(11), 1330–1334 (2000).
13. S. Cui, X. Zhu, W. Wang, and Y. Xie, "Calibration of a laser galvanometric scanning system by adapting a camera model," Appl. Opt. **48**(14), 2632–2637 (2009).
14. S. Zhang, and P. S. Huang, "Novel method for structured light system calibration," Opt. Eng. **45**(8), 083601 (2006).
15. R. L. Saenz, T. Bothe, and W. P. Juptner, "Accurate procedure for the calibration of a structured light system," Opt. Eng. **43**, 467–471 (2004).
16. O. Faugeras, "Three-Dimensional Computer Vision: A Geometric Viewpoint," (MIT Press, 1993).
17. C. Steger, "An Unbiased Detector of Curvilinear Structures," IEEE Trans. Pattern Anal. Mach. Intell. **20**(2), 113–125 (1998).
18. P. S. Huang, and S. Zhang, "Fast three-step phase-shifting algorithm," Appl. Opt. **45**(21), 5086–5091 (2006).

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

19. J. Meneses, T. Gharbi, and P. Humbert, "Phase-unwrapping algorithm for images with high noise content based on a local histogram," Appl. Opt. **44**(7), 1207–1215 (2005).

1. Introduction

Recently used calibration methods in optical triangular profilometry of diffuse surface can be classified into two categories based on whether a reference plane is used or not: the reference-plane approach and the no-reference-plane approach. One of the key features of the reference-plane approaches is the deduction of calibration equations from the geometry of the system. For example, Zhou *et al.* proposed a phase to height method based on the geometrics of the optical triangular profilometry in 1994 [1], where the heights (distances) between the test and reference planes are used in calibration procedure. They solve the problem of the co-plane for the camera axis and the projector axis and give the expression of the phase to height in two dimensions: $1/h(x,y) = p(x,y)/a(x,y) + b(x,y)$, parameters $a(x,y), b(x,y)$ are the coefficients at the object position (x,y). In their system, the camera must be in normal view of the reference plane. Chen *et al.* used a least-squares approach to evaluate the carrier phases [2, 3], and Guo *et al.* used similar approach in their calibration methods [4, 5], the cameras in their system setup are in normal view too, which is a rigorous condition difficult to satisfy in practice. Rajoub *et al.* proposed a method supporting the rotation of the camera [6], but they did not solve the problems of the normal view of the camera, either. Wang *et al.* proposed a method in which the camera can be in any view of the reference plane [7–9], but the expression of their conclusion is complicated and has lots of coefficients. All the above algorithms use the image ray tracing method, which cannot be used in the non-phase shift measuring profilometry. Anand *et al.* proposed the unified calibration techniques using the projection ray tracing method [10], but the setup of their system is restricted to rotating work table systems. It requires the normal incident of the projection line stripes too. The restrictions of the above-mentioned systems mainly stem from the use of geometry in their deduction, which also makes the process inefficient and tedious.

Instead of deducing the calibration equations from the geometry of the system, the no-reference-plane approach sets up a world coordinates system in which all the points in the measuring volumes are represented by the three-dimensional (3D) coordinates. The projector and the camera are always modeled as pin-hole camera models [11–15], and the intrinsic and extrinsic parameters can be calibrated by single device calibration techniques [11–14] or unite calibration techniques [15]. The process of obtaining calibration equations is significantly simplified and generalized as the systems are described by the transitions of the matrices and the rotation and translation effects are considered.

In this work, we use the matrices to describe the measuring system under reference-plane approach as an attempt to simplify and generalize the process of deducing the calibration equations. The camera and projector are modeled as pin-hole camera model, and the reference (test) planes are modeled by parallel planes. For the generalization, the direction of the camera, projector and reference (test) planes is in arbitrary position and direction in the world coordinates system. When the projection point and image point positions are known, the world coordinates of the point can be obtained by solving the equations formed by the system parameters. So the relationship of the world coordinate position, projection position and image position can be acquired. By deducing from this relationship, for both the projection ray and image ray tracing systems, the linear relationship of the reciprocal of the height and the reciprocal of the phase shift (or pixel position vertical distance) can be acquired.

The rest of the paper is organized as follows. In Section 2, after introducing the setup of the measuring system, the calibration method of projection ray tracing line stripe profilometry is deduced. It is then extended to the phase shift measuring profilometry. And then the calibration method of image ray tracing phase shift measuring profilometry is deduced to get simple calibration equation. The distortion effects of the camera and projector lenses are considered at the end of Section 2. In Section 3, experiments are conducted to verify the proposed method. The paper is concluded in Section 4.

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

2. Methods

2.1 Projection ray tracing line stripe system calibration

Figure 1 gives the schematics of the system setup. The reference plane S_{ref} is an arbitrary plane given by Eq. (1) in the world coordinate system $\{o^w; x^w, y^w, z^w\}$.

$$ax + by + cz + d_{ref} = 0. \quad (1)$$

$$ax + by + cz + d_i = 0. \quad (2)$$

$$\mathbf{v}_s = [a, b, c]^T. \quad (3)$$

The plane S_i, which parallels to the reference plane, is given by Eq. (2). The normal vector $\mathbf{v}_s$ of the planes is given by Eq. (3). So the height between surface S_i and S_{ref} can be given by following Eq. (4).

$$h_i = \left(d_i - d_{ref}\right) / \sqrt{a^2 + b^2 + c^2}. \quad (4)$$

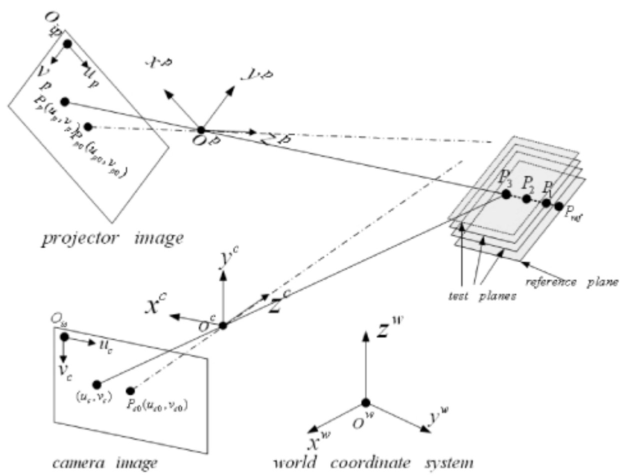

Fig. 1. Schematics of the system setup.

The camera and the projector are described by pin-hole camera model [11–13]. Point o^c is the optical lens center of the camera, and point o^p is the optical lens center of the projector. Camera image coordinate system $\{o_{ic}; u_c, v_c\}$ is constructed in the camera image sensor plane with the origin at left top, and the principal point is $P_{c0}\left(u_{c0}, v_{c0}\right)$. The camera device coordinate system $\{o^c; x^c, y^c, z^c\}$ is constructed with the origin at point o^c, with axis $o^c x^c$ parallels to axis $o_{ic} u_c$ and axis $o^c y^c$ parallels to axis $o_{ic} v_c$. Axis $o^c z^c$ is the main optical axis of the camera image lens. Through similar approach, the projector image coordinate system $\{o_{ip}; u_p, v_p\}$ and the projector device coordinate system $\{o^p; x^p, y^p, z^p\}$ are constructed, the principal point of the projector image plane is $P_{p0}\left(u_{p0}, v_{p0}\right)$. The distances between the image plane and the origin point are f_c (camera focus length) and f_p (projector focus length) for camera and projector systems respectively.

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

First of all, we will deduce the relationship of height d_i, projection position and image position. Assuming that $P_p(u_p, v_p)$ is an arbitrary point in the projector image plane, which is projected on the plane S_1 at point P_1 with the coordinate $\mathbf{X}_1 = [x_1, y_1, z_1]^T$ and on the reference plane S_{ref} at point P_{ref}. This projection procedure can be described by the following sequence of transformations: the coordinate $\mathbf{X}_1$ of world coordinate system is translated to projector device coordinate system coordinate $\mathbf{X}_p$ by Eq. (5),

$$\mathbf{X}_p = \mathbf{R}_p \mathbf{X}_1 + \mathbf{t}_p. \tag{5}$$

where $\mathbf{R}_p$ is a 3 × 3 orthogonal rotation matrix of the two coordinate systems, it is an expression of the rotation vector $\mathbf{r}_p = \left[r_{px}, r_{py}, r_{pz} \right]^T$, which parallels the rotation axis and whose magnitude is equal to the rotation angle. The relationship between $\mathbf{R}_p$ and $\mathbf{r}_p$ can be described by Rodrigues equation [16]. $\mathbf{t}_p = \left[t_{px}, t_{py}, t_{pz} \right]^T$ is a translation vector. The projector device coordinate $\mathbf{X}_p$ is translated to projector image plane by Eq. (6),

$$s_p \left[u_p \quad v_p \quad 1 \right]^T = \mathbf{P}_p \mathbf{X}_p. \tag{6}$$

where $\mathbf{P}_p = \begin{bmatrix} f_{pu} & 0 & u_{p0} \\ 0 & f_{pv} & v_{p0} \\ 0 & 0 & 1 \end{bmatrix}$, $f_{pu} = f_p / d_{pu}, f_{pv} = f_p / d_{pv}$, parameters d_{pu}, d_{pv} are the pixel sizes along horizontal and vertical direction of the projector image plane respectively, which are *the priori-knowledge*, and s_p is a scale factor. By substituting Eq. (5) to Eq. (6), Eq. (7) can be obtained.

$$s_p \left[u_p \quad v_p \quad 1 \right]^T = \mathbf{P}_p \left(\mathbf{R}_p \mathbf{X}_1 + \mathbf{T}_p \right). \tag{7}$$

Similarly, $\mathbf{X}_1$ can be translated to camera image coordinate (u_c, v_c) by following Eq. (8),

$$s_c \left[u_c \quad v_c \quad 1 \right]^T = \mathbf{P}_c \left(\mathbf{R}_c \mathbf{X}_1 + \mathbf{T}_c \right). \tag{8}$$

where $\mathbf{P}_c = \begin{bmatrix} f_{cu} & 0 & u_{c0} \\ 0 & f_{cv} & v_{c0} \\ 0 & 0 & 1 \end{bmatrix}$, $f_{cu} = f_c / d_{cu}, f_{cv} = f_c / d_{cv}$, parameters d_{cu}, d_{cv} are the pixel sizes along horizontal and vertical direction of the image plane respectively, which are *the priori-knowledge*, and s_c is a scale factor. $\mathbf{R}_c$ is the 3×3 orthogonal rotation matrix of the camera device coordinate system and world coordinate system, and $\mathbf{t}_c = \left[t_{cx}, t_{cy}, t_{cz} \right]^T$ is the translation vector. By defining $\mathbf{M}_p = \mathbf{P}_p \mathbf{R}_p$, $\mathbf{t}_{bp} = \mathbf{P}_p \mathbf{t}_p$, $\mathbf{M}_c = \mathbf{P}_c \mathbf{R}_c$, $\mathbf{t}_{bc} = \mathbf{P}_c \mathbf{t}_c$, Eq. (7) can be written as the following Eq. (9),

$$s_p \left[u_p \quad v_p \quad 1 \right]^T = \mathbf{M}_p \mathbf{X}_1 + \mathbf{t}_{bp}. \tag{9}$$

Equation (8) can be written as following Eq. (10).

$$s_c \left[u_c \quad v_c \quad 1 \right]^T = \mathbf{M}_c \mathbf{X}_1 + \mathbf{t}_{bc}. \tag{10}$$

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

Parameters $\mathbf{M}_c, \mathbf{M}_p$ are 3 × 3 matrices, for the convenience of the induction, let $\mathbf{M}_c = \left[\mathbf{m}_{c1}, \mathbf{m}_{c2}, \mathbf{m}_{c3}\right]^T$, $\mathbf{M}_p = \left[\mathbf{m}_{p1}, \mathbf{m}_{p2}, \mathbf{m}_{p3}\right]^T$, $\mathbf{m}_{ci}, \mathbf{m}_{pi}$ are the row vectors of matrices $\mathbf{M}_c, \mathbf{M}_p$. Similarly, let $\mathbf{t}_{bc} = \left[t_{bcx}, t_{bcy}, t_{bcz}\right]^T$, $\mathbf{t}_{bp} = \left[t_{bpx}, t_{bpy}, t_{bpz}\right]^T$. Once the system is set up, parameters $\mathbf{R}_c, \mathbf{R}_p, \mathbf{t}_p, \mathbf{t}_c, \mathbf{P}_p, \mathbf{P}_c$ are changeless, so parameters $\mathbf{M}_c, \mathbf{M}_p, \mathbf{t}_{bp}, \mathbf{t}_{bc}$ are changeless too. From Eq. (9) and Eq. (10), the following Eq. (11) can be constructed to acquire the world coordinate $\mathbf{X}_1$ when u_c, v_c, v_p are known.

$$\mathbf{A}\mathbf{X}_1 = \mathbf{B}. \tag{11}$$

where $\mathbf{A} = \begin{bmatrix} \mathbf{m}_{c1} - u_c\mathbf{m}_{c3} \\ \mathbf{m}_{c2} - v_c\mathbf{m}_{c3} \\ \mathbf{m}_{p2} - v_p\mathbf{m}_{p3} \end{bmatrix}, \mathbf{B} = -\begin{bmatrix} t_{bcx} - u_c t_{bcz} \\ t_{bcy} - v_c t_{bcz} \\ t_{bpy} - v_p t_{bpz} \end{bmatrix}$. Coordinate $\mathbf{X}_1$ is in plane S_1, substitute $\mathbf{X}_1$ and Eq. (3) to Eq. (2), the following Eq. (12) can be obtained.

$$\mathbf{v}_s^T\mathbf{X}_1 + d_i = 0. \tag{12}$$

d_i is given by following Eq. (13).

$$d_i = -\mathbf{v}_s^T\mathbf{X}_1. \tag{13}$$

By substituting the solution of Eq. (11) to Eq. (13), Eq. (14) can be obtained. The explicit expressions of Eq. (14) are calculated by symbol computation in MATLAB. In the calculation procedure, parameters $\mathbf{M}_c, \mathbf{M}_p, \mathbf{t}_{bp}, \mathbf{t}_{bc}, \mathbf{v}_s$ are constant, and u_c, v_c, v_p are variables. From the calculation result of Eq. (13), it can be found out that d_i is the sum of three fractions, of which denominator are the same, and both the numerator and denominator contain the variable items of $v_c, u_c, v_p v_c, v_p u_c, v_p$ and the constant item. Therefore, Eq. (14) can be obtained.

$$d_i = \frac{a_1 v_c + a_2 u_c + a_3 + a_4 v_p v_c + a_5 v_p u_c + a_6 v_p}{b_1 v_c + b_2 u_c + b_3 + b_4 v_p v_c + b_5 v_p u_c + b_6 v_p}. \tag{14}$$

where $a_1, a_2, a_3, a_4, a_5, a_6$, $b_1, b_2, b_3, b_4, b_5, b_6$ are constant parameters determined by system parameters $\mathbf{M}_p, \mathbf{M}_c, \mathbf{t}_{bp}, \mathbf{t}_{bc}$ and reference plane parameters a, b, c. To simplify the description, define $c_1 = (a_1 + a_4 v_p)/(b_1 + b_4 v_p)$, $c_2 = (a_2 + a_5 v_p)/(a_1 + a_4 v_p)$, $c_3 = (a_3 + a_6 v_p)/(a_1 + a_4 v_p)$, $c_4 = (b_2 + b_5 v_p)/(b_1 + b_4 v_p)$, $c_5 = (b_3 + b_6 v_p)/(b_1 + b_4 v_p)$, then Eq. (14) turns to Eq. (15),

$$d_i = \frac{k_{c1} u_c + k_{c2}}{v_c + c_4 u_c + c_5} + c_1. \tag{15}$$

where $k_{c1} = c_1(c_2 - c_4)$, $k_{c2} = c_1(c_3 - c_5)$.

Next, we will investigate the relationship between the height and image pixel vertical distance of the same projection points from Eq. (15) for the projection ray tracing line stripe pattern systems. Line stripe pattern $v_p = v_{pgiven}$ of the projection image, shown in Fig. 2(a), is projected on reference plane. The image is acquired by camera, which is shown in Fig. 2(b). As the surface plane S_i is placed ahead of the reference plane, the image acquired by camera is shown in Fig. 2(c). The two images are combined together for illustration, as shown in Fig. 2(d). For the two lines, the values of v_p are the same, so parameters $c_1, c_2, c_3, c_4, c_5, k_{c1}, k_{c2}$ are the same. By substituting v_{cref} to Eq. (15), parameter d_{ref} can be represented by the following Eq. (16):

$$d_{ref} = \frac{k_{c1}u_c + k_{c2}}{v_{cref} + c_4 u_c + c_5} + c_1. \tag{16}$$

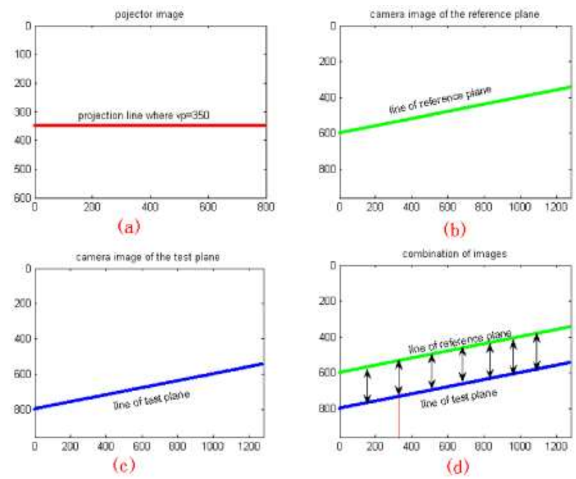

Fig. 2. Line stripe pattern of the projectors image and camera image. (a) A line stripe where $v_p = v_{pgiven}$ on the projector image. (b) An image of the reference plane. (c) An image of the test plane. (d) The combination of two images.

Equation (15) and Eq. (16) are substituted to Eq. (4), so Eq. (17) can be obtained.

$$h_i = \frac{k_{n1}u_c + k_{n2}}{v_c + c_4 u_c + c_5} - \frac{k_{n1}u_c + k_{n2}}{v_{cref} + c_4 u_c + c_5}. \tag{17}$$

where $k_{n1} = k_{c1}\sqrt{a^2 + b^2 + c^2}$, $k_{n2} = k_{c2}\sqrt{a^2 + b^2 + c^2}$. By calculating the reciprocal of Eq. (17), the following Eq. (18) can be obtained,

$$\frac{1}{h_i} = -\frac{(v_{cref} + c_4 u_c + c_5)^2}{k_{n1}u_c + k_{n2}} \frac{1}{v_c - v_{cref}} - \frac{v_{cref} + c_4 u_c + c_5}{k_{n1}u_c + k_{n2}}. \tag{18}$$

From Eq. (16), $v_{cref} + c_4 u_c + c_5 = (k_{c1}u_c + k_{c2}) / (d_{ref} - c_1)$ can be known, by substituting it to Eq. (18), Eq. (19) can be obtained.

$$\frac{1}{h_i} = -\frac{k_{c1}u_c + k_{c2}}{(d_{ref} - c_1)^2\sqrt{a^2 + b^2 + c^2}} \frac{1}{v_c - v_{cref}} - \frac{1}{(d_{ref} - c_1)\sqrt{a^2 + b^2 + c^2}}. \tag{19}$$

Define $y = h_i^{-1}$, $x = (v_c - v_{cref})^{-1}$, $v_c - v_{cref}$ is the vertical distance between the reference line and test lines, which is shown in Fig. 2(d), represented by the double row line, Eq. (19) turns to Eq. (20), parameter p_1 is given by Eq. (21),

$$y = p_1(u_c, v_c)x + p_2. \tag{20}$$

$$p_1(u_c, v_c) = k_{b1}u_c + k_{b2}. \tag{21}$$

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

where $k_{b1} = -k_{c1}(d_{ref} - c_1)^{-2}(a^2 + b^2 + c^2)^{-0.5}$, $k_{b2} = -k_{c2}(d_{ref} - c_1)^{-2}(a^2 + b^2 + c^2)^{-0.5}$, $p_2 = -(d_{ref} - c_1)^{-1}(a^2 + b^2 + c^2)^{-0.5}$ are constant for the stripe line. $p_1(u_c, v_c)$ is the coefficient at the camera image pixel position (u_c, v_c), it does not contain the item v_c. This equation can be used conveniently in optical stripe pattern measuring systems due to the projection ray tracing fact.

After the deduction of the calibration equation, the calibration procedure for the projection ray tracing line stripe profilometry can be described by the following steps:

step 1. Project the line stripe pattern to the reference planes and acquire the image (noted as reference image) by camera.

step 2. Place N(N≥2) parallel planes on top of the reference planes(noted as test plane 1, …,test plane N) and acquire image respectively(noted as test image 1,…, test image N).

step 3. Extract the line positions of the reference image and test images by algorithms, such as C. Steger method [17].

step 4. For a horizontal position u_c of the test images, calculate $v_{dif} = v_c - v_{cref}$ to form $\left[v_{dif1}, v_{dif2}, ..., v_{difN}\right]$, and the height between test planes and reference plane are $\left[h_1, h_2, ..., h_N\right]$, p_1, p_2 at horizontal position u_c can be determined by using the linear fitting algorithm of Eq. (20).

step 5. Repeat step 4 for every u_c value to form a coefficient table.

step 6. The u_c , p_1 values acquired in step 5 is fitted by linear algorithm of Eq. (21) to acquire coefficients k_{b1}, k_{b2}, this will reduce the noise influence(relative to the single point).

After the system is calibrated and parameters k_{b1}, k_{b2}, p_2 are determined, given an arbitrary specimen, its image is acquired by camera. After the extraction of the line position, the height can be calculated by Eq. (22).

$$h(u_c, v_c) = \left\{(k_{b1}u_c + k_{b2})(v_c - v_{cref})^{-1} + p_2\right\}^{-1}. \tag{22}$$

2.2 Projection ray tracing phase shift measuring profilometery calibration

The above method can also be extended to be used in the phase shift measuring profilometry [18], where v_p is a variable item. Since parameters k_{c1}, k_{c2}, c_1 are not constant, the coefficients of Eq. (20) at pixel position (u_c, v_c) turn to following Eq. (23) and Eq. (24),

$$p_1(u_c, v_c, v_p) = \frac{k_{p1a}v_p + k_{p1b}}{(v_p + k_{p0})^2}u_c + \frac{k_{p2a}v_p + k_{p2b}}{(v_p + k_{p0})^2}. \tag{23}$$

$$p_2(u_c, v_c, v_p) = (k_{pa}v_p + k_{pb}) / (v_p + k_{p0}). \tag{24}$$

v_p can be translated from the phase by the following Eq. (25),

$$v_p = \phi / 2\pi. \tag{25}$$

Parameters $k_{p0}, k_{p1a}, k_{p1b}, k_{p2a}, k_{p2b}, k_{pa}, k_{pb}$ are the functions of $a_1, a_2, a_3, a_4, a_5, a_6$, b_1, b_2, b_3, b_4 , b_5, b_6 . The key point of the algorithms is to determine the

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

reference line of the given phase value ϕ. For a given pixel position (u_{c1}, v_{c1}) of the test plane image (noted as image 1), its extraction phase is ϕ_1, to calculate the height by Eq. (22), the pixel position (u_{c1ref}, v_{c1ref}) of the same phase ϕ_1 on the reference plane image (noted as image 2) should be found out. Then the relationship of v_{c1} and v_{p1} will be discussed. Define

$\mathbf{D} = \begin{bmatrix} \mathbf{m}_{c1} - u_{c1}\mathbf{m}_{c3} \\ \mathbf{m}_{p2} - v_{p1}\mathbf{m}_{p3} \\ a \quad b \quad c \end{bmatrix}$, $\mathbf{E} = -\begin{bmatrix} t_{bcx} - u_{c1}t_{bcz} \\ t_{bpy} - v_{p1}t_{bpz} \\ d_1 \end{bmatrix}$, so the Point P_i's coordinates can be got from equation $\mathbf{D}\mathbf{X}_i = \mathbf{E}$, this equation is constructed from Eq. (2), Eq. (9) and Eq. (10). Then the image point of $\mathbf{X}_i$ in the camera can be calculated from Eq. (10). By substituting the solution of equation $\mathbf{D}\mathbf{X}_i = \mathbf{E}$ to Eq. (10), Eq. (26) can be obtained. The explicit expression of Eq. (26) is obtained by the symbol computing in Matlab. In the calculation procedure, parameters $\mathbf{M}_p, \mathbf{M}_c, \mathbf{t}_{bp}, \mathbf{t}_{bc}$, a, b, c, d_1 and u_{c1} are constant, the only variable is v_{p1}. The calculation result is in the form of fraction after the simplification, where both the numerator and denominator contain single variable item v_{p1}. So it can be written in the form of Eq. (26).

$$v_{c1} = \frac{k_{rc1}}{v_{p1} + k_{rc0}} + k_{rc2}. \qquad (26)$$

Parameters $k_{rc1}, k_{rc2}, k_{rc0}$ are determined by system constant parameters $\mathbf{M}_p, \mathbf{M}_c, \mathbf{t}_{bp}, \mathbf{t}_{bc}$, a, b, c, d_i and parameter u_c (known value).

After the deduction of the calibration equation, the calibration procedure of projection ray tracing phase shift measuring profilometery can be described by the following steps:

step 1. The phase shift images are projected on the reference planes and test planes, the images are acquired by camera.

step 2. The phase are calculated and unwrapped for each plane (noted as reference phase, test phase 1, …, test phase M) [19].

step 3. For each column pixels of the reference phase, variable v_{c1ref}, v_{p1} are known, so nonlinear fit algorithm can be used to determine the parameters $k_{rc1}, k_{rc2}, k_{rc0}$ in Eq. (26).

step 4. Repeat step 3 for each test phases.

step 5. For one point (u_{ck}, v_{cki}) in test plane i, its phase is ϕ_t ,and corresponding projection image position is v_{p1}. By taking the parameters obtained in step3, step4, the reference pixel position (u_{ck}, v_{ckref}), the corresponding pixel positions (u_{ck}, v_{ck1}), …, (u_{ck}, v_{ckN}) of the same phase on other test planes can be determined through Eq. (26), parameters p_1, p_2 can be determined by linear fit algorithms of Eq. (20).

step 6. Repeat step 5 for each points of the test plane i, parameters $p_1(u_{c1}, v_{c1}, v_{p1})$, $p_1(u_{c2}, v_{c2}, v_{p2})$, …, $p_1(u_{cM}, v_{cM}, v_{pM})$ and $p_2(u_{c1}, v_{c1}, v_{p1})$, $p_2(u_{c2}, v_{c2}, v_{p2})$,…, $p_2(u_{cM}, v_{cM}, v_{pM})$ are acquired, and $u_{c1}, v_{c1}, v_{p1}, u_{c2}, v_{c2}, v_{p2}, ..., u_{cM}, v_{cM}, v_{pM}$ are known, so lest square methods can be used to determine the parameters $k_{p0}, k_{p1a}, k_{p1b}, k_{p2a}, k_{p2b}, k_{pa}, k_{pb}$ in Eq. (23) and Eq. (24).

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

After the calibration is done, parameters can be used in the measure procedure. The phase image on the specimen is computed. For each pixel (u_c, v_c), its phase ϕ is translated to v_p by Eq. (25), and then the reference pixel position is acquired by Eq. (26). p_1, p_2 are determined by Eq. (23), Eq. (24), so height can be determined by Eq. (27).

$$h = \left\{ p_1(u_c, v_c, v_p)(v_c - v_{cref})^{-1} + p_2(u_c, v_c, v_p) \right\}^{-1} \tag{27}$$

2.3 Image ray tracing phase shift measuring profilometery calibration

From the above description of Section 2.2, it can be known that the calibration and calculation procedure is complicated for the phase shift measuring profilometry, and it's not convenient, particularly in finding the reference position of given phase. If the image ray tracing method is used, we can use the phase shift information at the same pixel position for calibration. It's convenient for calculating. Rewrite Eq. (14) as following Eq. (28),

$$d_i = t_1 + \frac{t_2}{v_p + t_3}. \tag{28}$$

where $t_1 = (a_4 v_c + a_5 u_c + a_6)(b_4 v_c + b_5 u_c + b_6)^{-1}$, $t_2 = \left\{(a_1 v_c + a_2 u_c + a_3)(b_4 v_c + b_5 u_c + b_6) - (b_1 v_c + b_2 u_c + b_3)(a_4 v_c + a_5 u_c + a_6)\right\}(b_4 v_c + b_5 u_c + b_6)^{-2}$, $t_3 = (b_1 v_c + b_2 u_c + b_3)(b_4 v_c + b_5 u_c + b_6)^{-1}$. For the same pixel position (u_c, v_c) of the camera image plane, parameters t_1, t_2, t_3 are constant, so Eq. (19) turns to Eq. (29).

$$\frac{1}{h_i} = -\frac{1}{d_{ref} - t_1} - \frac{t_2}{(d_{ref} - t_1)^2} \frac{1}{v_p - v_{pref}}. \tag{29}$$

This equation equals to Eq. (20) when $p_1 = -t_2(d_{ref} - t_1)^{-2}, p_2 = -(d_{ref} - t_1)^{-1}$. v_p, v_{pref} can be translated from the phase by Eq. (25). For each point, the reciprocal of the height and reciprocal of the phase shift satisfy Eq. (29).

Then the calibration procedure of image ray tracing phase shift measuring profilometery calibration can be described by the following steps:

step 1. The phase shift images are projected on the reference planes and test planes, the images are acquired by camera.

step 2. The phase are calculated and unwrapped for each plane (noted as reference phase, test phase 1, …, test phase M).

step 3. For pixel position (u_{ck}, v_{ck}), the reference phase is ϕ_{ref}, its corresponding v_p value is v_{pref}, for test planes, v_p values are v_{p1}, v_{p2}, …, v_{pM}. Height h_i^{-1} is known and $(v_{pi} - v_{pref})^{-1}$ is computed, so parameters p_1, p_2 are determined by linear fit algorithm of Eq. (20);

step 4. Repeat step 3 for each pixel position of reference image, the coefficients table for p_1, p_2 can be formed.

After calibration, the height can be calculated by Eq. (30).

$$h_i = \left\{ p_1(u_c, v_c)(v_{pi} - v_{pref})^{-1} + p_2(u_c, v_c) \right\}^{-1}. \tag{30}$$

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

2.4 Lens distortion compensation

All the above calculations base on the ideal model of the projector and camera lenses, but in the practical systems, the lens distortions of camera and the projector should be considered to achieve high measurement accuracies. The distortions of the lens usually contain radial and tangential distortion modeled by the following Eq. (31) [11].

$$\begin{aligned} u_d &= u + 2P_1uv + P_2(r^2 + 2u^2) + K_1ur^2 + K_2ur^4 + K_3ur^6 \\ v_d &= v + 2P_2uv + P_1(r^2 + 2v^2) + K_1vr^2 + K_2vr^4 + K_3vr^6. \end{aligned} \tag{31}$$

where (u, v) is the ideal point, and (u_d, v_d) is the distortion point, $r = \sqrt{u^2 + v^2}$, K_1, K_2 and K_3 are the coefficients of radial distortion, P_1 and P_2 are the coefficients of tangential distortions. Calibration procedure [11–13] can be used to acquire these distortion coefficients of the camera lens, and the projector lens distortions can be calibrated by the modify calibration procedure of Zhang *et al.* [14].

After the distortion coefficients having been calibrated, iteration method will be used to calculate the ideal point (u, v) from (u_d, v_d) for the camera image [15]. For the projector image, the ideal point (u, v) is the known value, and then the distortion point (u_d, v_d) is calculated by Eq. (31). When (u_d, v_d) is projected on the test object, after the distortion of the projector lens, the approximate ideal point can be acquired on the test objects. In the proposed method of this work, all the images acquired by the camera are compensated before being used in the calibration procedure, and all the projector images are compensated before being projected.

3. Experiments

We conduct experiments to verify the proposed method. The line stripe pattern optical triangular measuring system is set up as shown in Fig. 1. The reference plane, the camera and projector are fixed after being set up. Blocks of the fixed thickness are placed on the reference plane to form the parallel test planes. The image of reference plane acquired by the camera is shown in Fig. 3(a), one of the test images is shown in Fig. 3(b), and Fig. 3(c) are the combination of the reference and test line stripes extracted by Steger line position extraction algorithm from the images. By taking the procedures described in Section 2.1, parameters p_1, p_2 of each point can be determined. Figure 3(d) shows the plot of p_1 and u_c . From the figure, it can be found out that p_1 is linear with u_c suggesting that the experiment result coincides with the Eq. (21). After the calibration being done, a block with thickness of 162.3um is measured, the results are shown in Fig. 3(e), and the errors are shown in Fig. 3(f). The mean error is 0.03um and root mean square error is 0.21um. In the experiment, the camera and projector are in arbitrary position and direction and the reference plane is in arbitrary direction too.

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

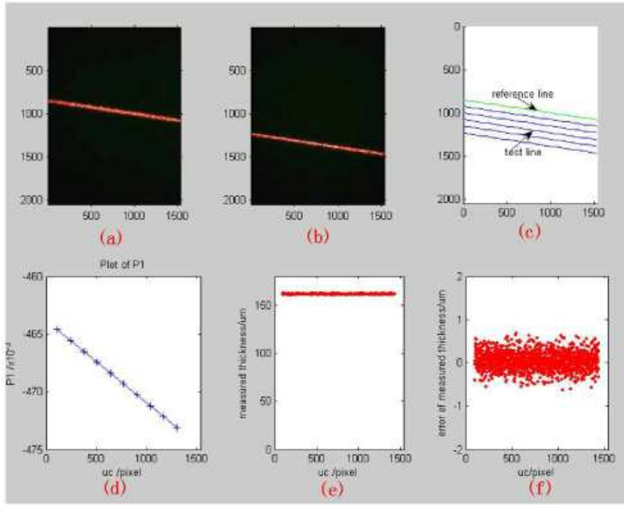

Fig. 3. Experiments result for the line stripe pattern profilometry. (a) Camera image of the reference plane. (b) Camera image of one of test planes. (c) The combination image of the extracted test and reference lines. (d) Plot of parameter p_1 and variable u_c . (e) Measuring result of a given block. (f) Error of (e).

A phase shift measuring profilometry experiment is conducted to verify the proposed image ray tracing calibration method. Three steps shift algorithm is used [18], the projector is NEC VT491 + digital projector, and the camera is SONY A-300, five test planes parallel to the reference plane are used, the distance from the reference plane are 200mm, 400mm, 600mm, 800mm, and 1000mm, the distance accuracy is ± 0.05mm, and the camera and projector are focused at the distance of 500 mm from the reference plane. The system is calibrated by procedures described in Section 2.3, and a coefficients table for each pixel position (u_c, v_c) is acquired. These parameters are used to calculate the height of a given plane whose distance from the reference plane is 500mm, the measured distance is shown in Fig. 4(a), the errors are shown in Fig. 4(b), the mean distance value of the measured plane is 499.92mm, and the root mean square error is 0.12mm. A face experiment is also conducted to verify the method for the complicated objects. Figure 5(a) shows 2D form of the measured face, the height of the face is shown as gray values, and Fig. 5(b) shows the 3-D mesh grid of the measured face.

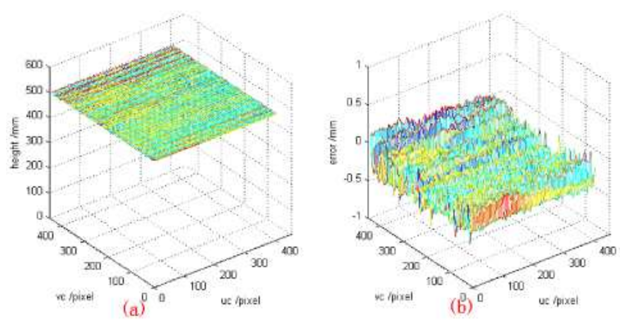

Fig. 4. The measured result of the given plane. (a) Measured result. (b) The error of the measured result

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009

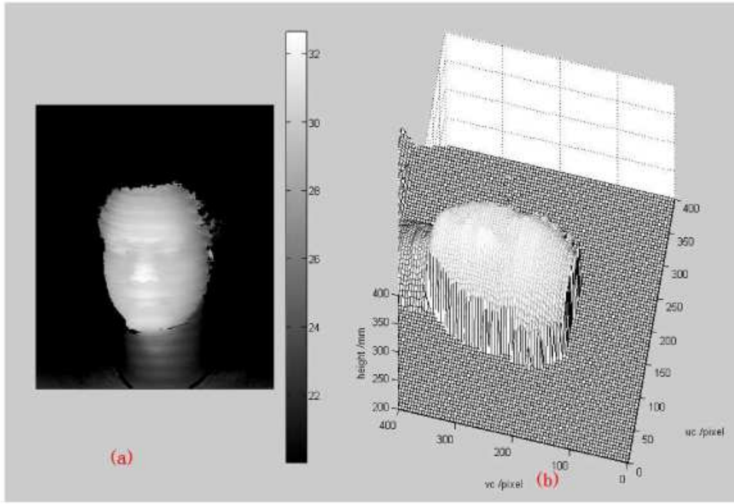

Fig. 5. Measured result of face. (a) 2D show of the face, gray value represented the height dedicated by the color bar on the right side of the Fig. 5(b) 3D mesh grid of the measured face.

4. Conclusion

In this work, we propose a generalized calibration method in optical triangular profilometry by modeling the projector and camera as pin-hole camera models and the reference (test) planes as parallel planes. For the projection ray tracing line strip measuring systems, the reciprocal of height and the reciprocal of the pixel position vertical distance are in linear relationship, and for the image ray tracing phase shift measuring profilometry, the reciprocal of height and the reciprocal of the phase shift (change) of the same pixel position are in linear relationship. So for both the projection and image ray tracing systems, the simple linear calibration equation is obtained, which has fewer coefficients, so computer speed is fast for measuring. The distortion effects of images should be compensated before images being used in the proposed method. Experiments of the line stripe pattern and phase shift measuring profilometry are conducted to verify the proposed methods.

The calibration equations we obtained are consistent with the results from the geometry approaches. While the accuracy of the calibration was not significantly improved in our work as compared to the geometry approach [1–10], our work is the first attempt to use matrices in the reference-plane approaches. With the use of matrices, our model has a number of important advantages over the traditional model derived from geometry: firstly, the deduction of the calibration is more convenient and simple, giving results that are easy to understand; secondly, it supports the complicated rotations and translations for the system setup; thirdly, the same linear calibration equation for both projection and image ray tracing system can be acquired.

There are some extended results for this paper, for the image ray tracing phase shift measuring profilometry, the coefficients can be also given by the expression of parameters $a_1, a_2, a_3, a_4, a_5, a_6, b_1, b_2, b_3, b_4, b_5, b_6$, which are determined by the system setup. The coefficients table and (u_c, v_c) values can be used to determine these parameters by least square methods [7–9], but it is complicated and unnecessary in practice.

#116567 - $15.00 USD Received 1 Sep 2009; revised 21 Oct 2009; accepted 22 Oct 2009; published 28 Oct 2009